MATH 571: MATHEMATICAL LOGIC
HOMEWORK SET 4, DUE AT 8:50 ON FRIDAY, OCT. 2

BRING YOUR SOLUTIONS TO CLASS, OR SLIDE THEM UNDER THE DOOR OF VAN VLECK 403

1. Exercise 1.5.1 from Enderton.

2. Show that the set $\{\vee, \wedge\}$ is not complete. (Hint: show, using induction, that for any wff α only containing the connectives $\vee$ and $\wedge$, we have that $B^n_\alpha(T, \ldots, T) = T$).

3. Let Δ be such that:
   (1) For every finite subset $\Gamma \subseteq \Delta$, the set Γ is satisfiable.
   (2) For every formula φ, if every finite subset Γ of $\Delta \cup \{\varphi\}$ is satisfiable, then $\varphi \in \Delta$.

   Prove that for all wffs φ, ψ: $\varphi \vee \psi \in \Delta$ if and only if $[\varphi \in \Delta$ or $\psi \in \Delta]$. This completes the proof of the compactness theorem in class.

   (Hint: the left-to-right direction is the hardest one. For this direction, if both $\varphi, \psi \notin \Delta$, there are finite $\Gamma \subseteq \Delta \cup \{\varphi\}$ and $\Gamma' \subseteq \Delta \cup \{\psi\}$ which are not satisfiable; now consider $(\Gamma \cup \Gamma' \cup \{(\varphi \vee \psi)\}) \setminus \{\varphi, \psi\}$.

4. Exercise 1.7.4 from Enderton. Here, a coloring of a map is an assignment which assigns each country a color in such a way that no neighboring countries share the same color. You may use the fact that every finite map can be colored with 4 colors without proof (this proof is, in fact, not easy).