MATH 571: MATHEMATICAL LOGIC
HOMEWORK SET 8, DUE AT 8:50 ON FRIDAY, OCT. 30

BRING YOUR SOLUTIONS TO CLASS, OR SLIDE THEM UNDER THE DOOR OF VAN VLECK 403

1. Give proof trees which show the following:
   a. $\vdash \forall x \forall y \varphi \rightarrow \forall y \forall x \varphi$.
   b. $\vdash \exists x \exists y \varphi \rightarrow \exists y \exists x \varphi$.

2. Give proof trees which show the following:
   a. $\vdash \forall x(\varphi \wedge \psi) \rightarrow ((\forall x \varphi) \wedge (\forall x \psi))$.
   b. $\vdash ((\forall x \varphi) \wedge (\forall x \psi)) \rightarrow \forall x(\varphi \wedge \psi)$.

3. a. Give a proof tree which shows that $\vdash \exists x(\varphi \wedge \psi) \rightarrow ((\exists x \varphi) \wedge (\exists x \psi))$.
   b. Show that $\nvdash ((\exists x \varphi) \wedge (\exists x \psi)) \rightarrow \exists x(\varphi \wedge \psi)$. For this, you may use the completeness theorem, and therefore it is enough to give an example of two formulas φ, ψ and a model $\mathcal{A}$ such that $\mathcal{A} \nvDash ((\exists x \varphi) \wedge (\exists x \psi)) \rightarrow \exists x(\varphi \wedge \psi)$.

4. a. Give a proof tree which shows that $\vdash \exists x \forall y \varphi \rightarrow \forall y \exists x \varphi$.
   b. Show that $\nvdash \forall y \exists x \varphi \rightarrow \exists x \forall y \varphi$, again by giving an example of a formula φ and a model $\mathcal{A}$ such that $\mathcal{A} \nvDash \forall y \exists x \varphi \rightarrow \exists x \forall y \varphi$.